STAT 4630 Spring 2011 Final Exam

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Name:
PID:

Instructions

Read each problem carefully. Show all your work. Credit will only be awarded if your work is included.

(15pts)**Problem 1.** Consider two MA(2) processes, one with $\theta_1 = \theta_2 = 1/6$ and another with $\theta_1 = -1$ and $\theta_2 = 6$.

a. Show that these processes have the same autocorrelation function.

b. How do the roots of the corresponding characteristic polynomials compare?

c. Which one gives a stationary and invertible MA(2) process?

(40pts)**Problem 2.** For each of the following ARIMA(p,d,q) model, what are the values of p, d and q. Furthermore, state whether the model is stationary and/ or invertible. Explain your answer briefly. (It suffices to verify the conditions for stationarity and invertibility for the models.)

a. $Y_t = Y_{t-1} - 0.25Y_{t-2} + e_t - 0.3e_{t-1}$.

b. $Y_t = 10 + 1.25Y_{t-1} - 0.25Y_{t-2} + e_t - 0.1e_{t-1}$.

c. $Y_t = 5 + 2Y_{t-1} - 1.7Y_{t-2} + 0.7Y_{t-3} + e_t - 0.5e_{t-1} + 0.25e_{t-2}$

d. $Y_t = 0.25Y_{t-2} + e_t + 0.5e_{t-1}$.

e. $Y_t = 0.5Y_{t-1} + 0.5Y_{t-2} + e_t + 2e_{t-1}$

(20pts)**Problem 3.** Consider two models:

(1)

$A: Z_t = e_t - 0.3e_{t-1} - 0.15e_{t-2} - 0.075e_{t-3} - 0.0375e_{t-4} - 0.01875e_{t-5} - 0.009375e_{t-6}$

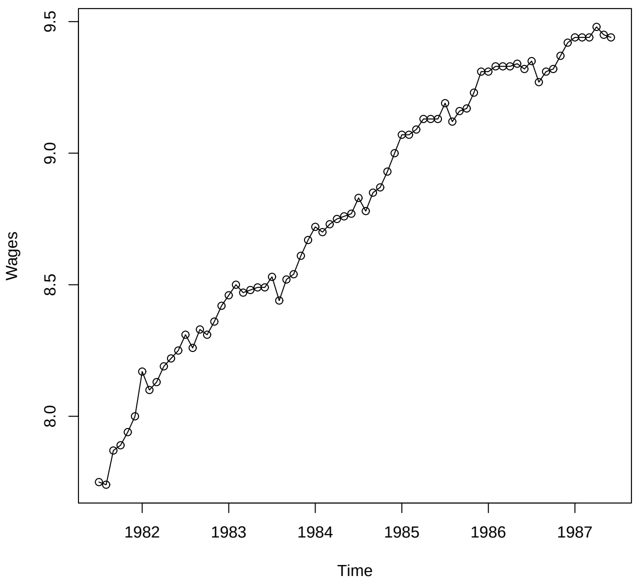
$B: Z_t - 0.5Z_{t-1} = e_t - 0.8e_{t-1}$

a.Find the ψ weights of the two models.

b. Are these two models similar? Explain your answer.

(40pts)**Problem 4.** For a time series of monthly values of average hourly wages (in dollars) for workers in the U.S. apparel and textile products industry for July 1981 through June 1987 is studied below.

a. Interpret the time series plot of this data

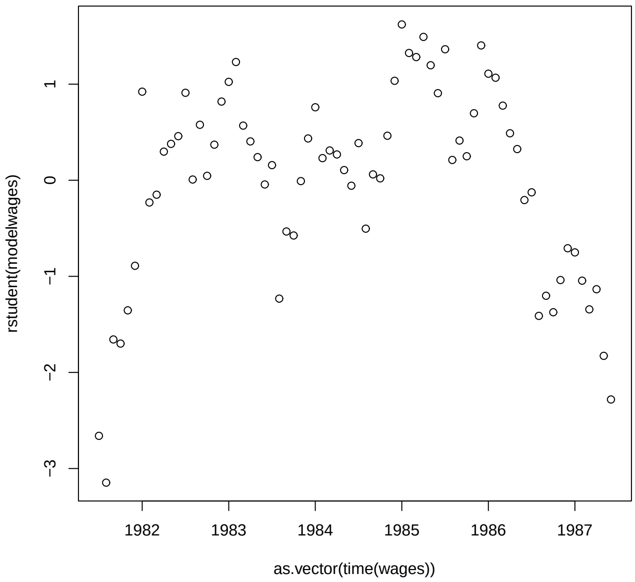


b. A least squares is used to fit a **linear time trend** to this time series. Interpret the regression output.

```
Call:
lm(formula = wages ~ time(wages))

Residuals:
     Min       1Q   Median       3Q      Max
-0.23828 -0.04981  0.01942  0.05845  0.13136

Coefficients:
              Estimate Std. Error t value Pr(>|t|)
(Intercept) -5.490e+02  1.115e+01  -49.24   <2e-16 ***
time(wages)  2.811e-01  5.618e-03   50.03   <2e-16 ***
---
Signif. codes:  0 *** 0.001 ** 0.01 * 0.05 . 0.1   1

Residual standard error: 0.08257 on 70 degrees of freedom
```

```
Multiple R-squared: 0.9728,     Adjusted R-squared: 0.9724
F-statistic:  2503 on 1 and 70 DF,  p-value: < 2.2e-16
```

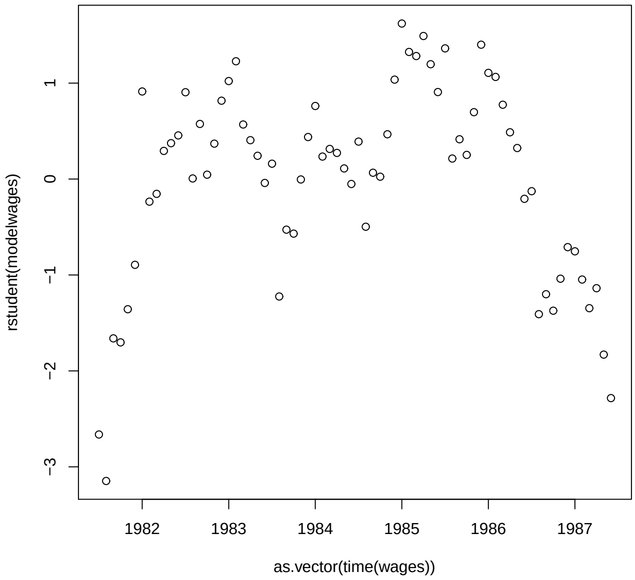
c. Interpret the time series plot of the standardized residuals from part **b.**



d. A least squares is used to fit a **quadratic time trend** to this time series. Interpret the regression output.

```
Call:
lm(formula = wages ~ time(wages) - 1 + I(time(wages)^2))

Residuals:
     Min       1Q   Median       3Q      Max
-0.23909 -0.04965  0.01972  0.05869  0.13175

Coefficients:
                  Estimate Std. Error t value Pr(>|t|)
time(wages)      -2.722e-01  5.637e-03  -48.29   <2e-16 ***
I(time(wages)^2)  1.394e-04  2.840e-06   49.08   <2e-16 ***
---
Signif. codes:  0 *** 0.001 ** 0.01 * 0.05 . 0.1   1

Residual standard error: 0.08283 on 70 degrees of freedom
Multiple R-squared: 0.9999,     Adjusted R-squared: 0.9999
F-statistic: 4.063e+05 on 2 and 70 DF,  p-value: < 2.2e-16
```

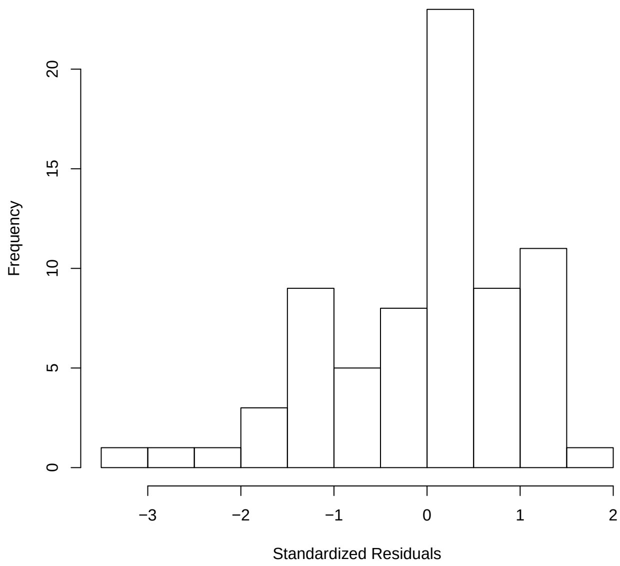
e. Interpret the time series plot of the standardized residuals from part **c.**

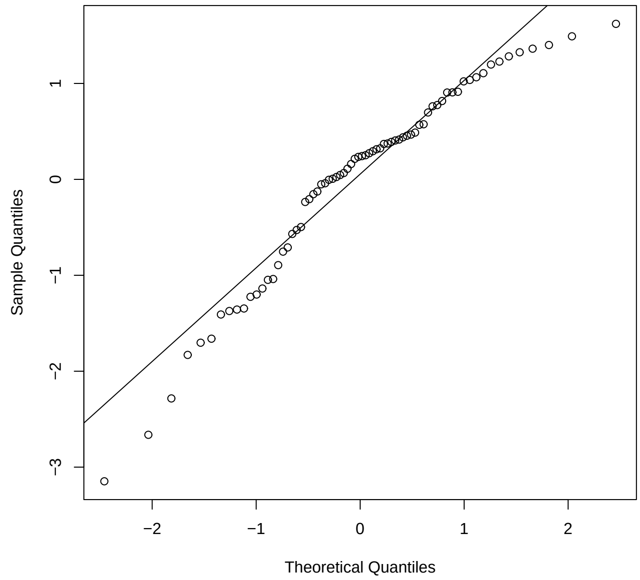


f. The output for the **runs test and the Shapiro-Wilk test for quadratic time trend** is as follows. Interpret the results

```
   $pvalue
[1] 2.55e-08

$observed.runs
[1] 13

$expected.runs
[1] 35.22222

$n1
[1] 28

$n2
[1] 44

$k
[1] 0


 Shapiro-Wilk normality test

data:  rstudent(modelwages)
W = 0.9428, p-value = 0.002612
```

g. Below are some model diagnostics performed on the residuals of **for quadratic time trend**. Comment on the model fit





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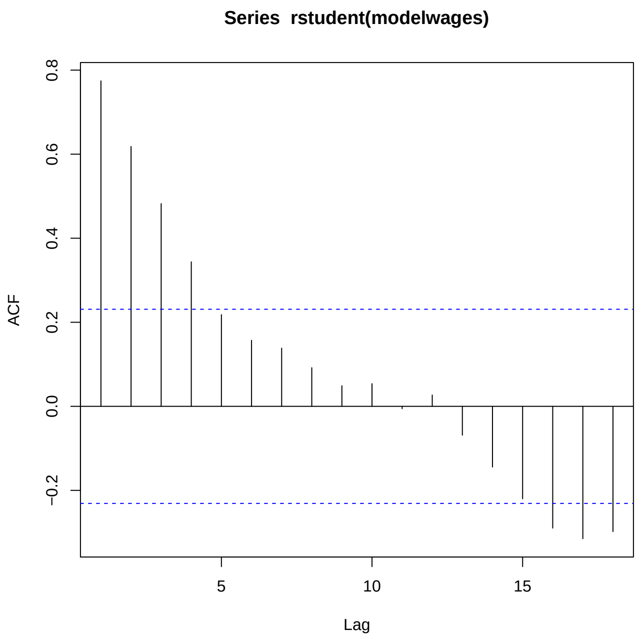
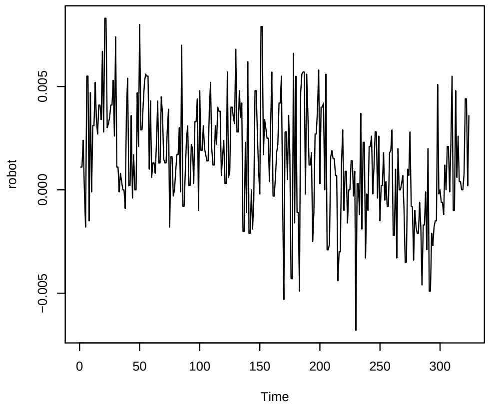
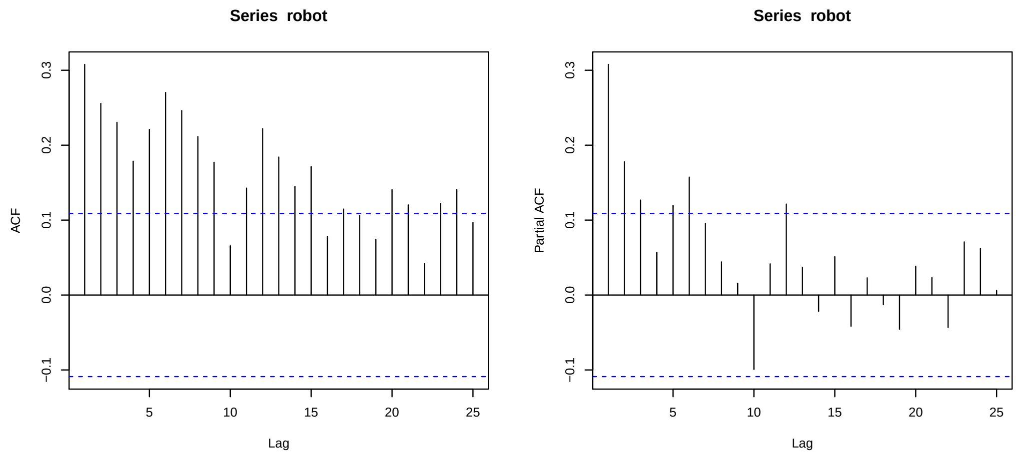


(40pts) **Problem 5.** Answer the following questions about a time series obtained from an industrial robot. The robot was put through a sequence of maneuvers, and the distance from a desired ending point was recorded in inches. This was repeated 324 times to form the time series. Look at the output from "R" to answer the questions

a. The time series is plotted. Do these data appear to come form a stationary or nonstationary process?



b. The ACF and PACF plots are given below. Given the plots, do these data appear to come from a stationary or a nonstationary process?



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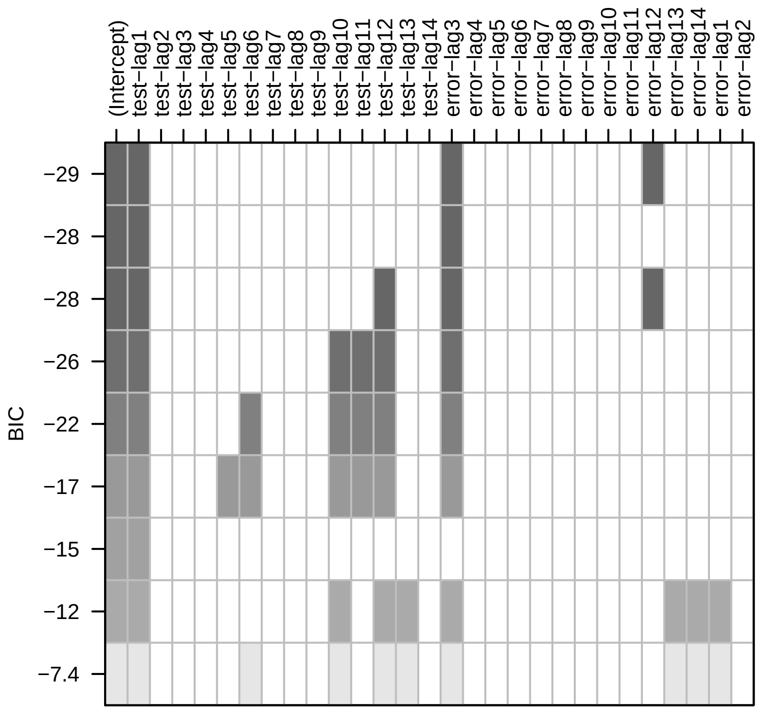
c. The sample EACF is given below. Interpret it.

AR/MA	0	1	2	3	4	5	6	7	8	9	10	11	12	13
0	x	x	x	x	x	x	x	x	x	o	x	x	x	x
1	x	o	o	o	o	o	o	o	o	o	o	o	o	o
2	x	x	o	o	o	o	o	o	o	o	o	o	o	o
3	x	x	o	o	o	o	o	o	o	o	o	o	o	o
4	x	x	x	x	o	o	o	o	o	o	o	o	x	o
5	x	x	x	o	o	o	o	o	o	o	o	o	x	o
6	x	o	o	o	o	x	o	o	o	o	o	o	o	o
7	x	o	o	x	o	x	x	o	o	o	o	o	o	o

d. The ARMASUBSETS output from "R" is given below. What is the best model you would specify with this information? Compare this results with what you observed in parts **a., b.**, and **c.**



(30pts)**Problem 6.** Consider the same robot data as in Problem 5.

a. The "R" output for the Dickey-Fuller test on the series with $k = 0$ is given. Comment on the results.

```
  --------- ------ - ------ ----
  Augmented Dickey & Fuller test
  --------- ------ - ------ ----

  Null hypothesis: Unit root.
  Alternative hypothesis: Stationarity.

----
  ADF statistic:

        Estimate Std. Error t value Pr(>|t|)
adf.reg   -0.692      0.053 -13.011     0.01

  Lag orders: 0
  Number of available observations: 323
Warning message:
In interpolpval(code = code, stat = adfreg[, 3], N = N) :
  p-value is smaller than printed p-value
```

b. The software "R" estimated the AR order to be $k = 7$. The "R" output for the Dickey-Fuller test on the series with $k = 7$ is given. Comment on the results.

```
   --------- ------ - ------ ----
  Augmented Dickey & Fuller test
  --------- ------ - ------ ----

  Null hypothesis: Unit root.
  Alternative hypothesis: Stationarity.

----
  ADF statistic:

        Estimate Std. Error t value Pr(>|t|)
adf.reg   -0.294      0.087   -3.39    0.013

  Lag orders: 1 2 3 4 5 6 7
  Number of available observations: 316
```

c. The parts **a.** and **b.** are repeated below for the differences of the robot data. Comment on the results.

```
 --------- ------ - ------ ----
  Augmented Dickey & Fuller test
  --------- ------ - ------ ----

  Null hypothesis: Unit root.
  Alternative hypothesis: Stationarity.

----
  ADF statistic:

        Estimate Std. Error t value Pr(>|t|)
adf.reg   -1.462       0.05 -29.436     0.01

  Lag orders: 0
  Number of available observations: 322
Warning message:
```

```
In interpolpval(code = code, stat = adfreg[, 3], N = N) :
  p-value is smaller than printed p-value



 --------- ------ - ------ ----
  Augmented Dickey & Fuller test
  --------- ------ - ------ ----

  Null hypothesis: Unit root.
  Alternative hypothesis: Stationarity.

----
  ADF statistic:

        Estimate Std. Error t value Pr(>|t|)
adf.reg   -5.304       0.66  -8.042     0.01

  Lag orders: 1 2 3 4 5 6 7 8 9 10 11
  Number of available observations: 311
Warning message:
In interpolpval(code = code, stat = adfreg[, 3], N = N) :
  p-value is smaller than printed p-value
```

(15pts)**Problem 7.** The sample ACF and PACF for a series are given in the following table. Here $n = 102$

lag	1	2	3	4	5	6	7
ACF for Yt	0.662	0.458	0.235	0.174	0.010	-0.139	-0.211
PACF for Yt	0.662	0.034	-0.144	0.109	-0.195	-0.178	0.021

Based on this information alone, which ARMA model(s) would we consider for the series?