

## STAT 4630 Spring 2011 Exam #2

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Name:

PID:

### Instructions

Read each problem carefully. Show all your work. Credit will only be awarded if your work is included.

Below,  $\{e_t, t = 0, 1, 2, \dots\}$  denotes a sequence of iid  $N(0, \sigma^2 = 1)$  random variables.

(20)**Problem 1.** Let  $X_t$  be an MA(1) process:  $X_t = e_t - \theta e_{t-1}$ . Let  $Y_t = X_t + X_{t-1}$ ,  $t = 0, 1, 2, \dots$ .

**a.** Show that  $Y_t$  is an MA(2) process.

**b.** Suppose that  $\theta = 0.4$ . Compute the autocorrelation function of  $Y_t$ .

(30pts)**Problem 2.** For each of the following ARIMA(p,d,q) model, what are the values of  $p, d$  and  $q$ . Furthermore, state whether the model is stationary and/ or invertible. Explain your answer briefly. (It suffices to verify the conditions for stationarity and invertibility for the models.)

a.  $Y_t = 5 + e_t - 1.5e_{t-1} + 0.5e_{t-2}$ .

b.  $Y_t = 1 + Y_{t-1} - 0.5Y_{t-2} + e_t - 0.5e_{t-1}$ .

c.  $Y_t = Y_{t-1} + e_t - 0.9e_{t-1}$

d.  $Y_t = 1 + \frac{5}{6}Y_{t-1} - \frac{1}{6}Y_{t-2} + e_t + 2e_t$ .

e.  $Y_t = 0.5Y_{t-1} + 0.5Y_{t-2} + e_t + 0.5e_{t-1}$

(25pts)**Problem 3.** Consider the two models:

$$(1) \quad \begin{aligned} A : Z_t + 1.3Z_{t-1} + 0.65Z_{t-2} + 0.325Z_{t-3} + 0.1625Z_{t-4} + 0.05Z_{t-5} &= e_t \\ B : Z_t + 0.8Z_{t-1} &= e_t - 0.5e_{t-1} \end{aligned}$$

**a.** Find the  $\pi$  weights of the two models.

**b.** Are these two models similar? Explain your answer.

(25pts)**Problem 4.** Suppose that  $Y_t = Y_{t-1} - 1 + e_t - 0.5e_{t-1}$ , where  $Y_0 = 1$ .

**a.** Show that  $\mu_t - \mu_{t-1} = -1$  for  $t \geq 1$  where  $\mu_t = E(Y_t)$ . Hence, or otherwise, find the mean of  $Y_t$  for  $t \geq 0$ . Does  $\mu_t = E(Y_t)$  converge to a finite limit as  $t \rightarrow \infty$ ?

**b.** Consider another model:  $Z_t = 0.95Z_{t-1} - 1 + e_t - 0.5e_{t-1}$ , where  $Z_0 = 1$ . Explain briefly why  $\mu_t = E(Z_t)$  converges to a limit as  $t \rightarrow \infty$ . Find the limit.