STAT 3600-096 Spring 2010 Quiz #4

Name:
PID:

(10pts)**Problem 1.** Suppose that the cumulative distribution function of X is given by

$$F(x) = 1 - x^{-6},\ 1 < x < \infty.$$

a. Describe how an observation of X can be simulated using a uniform $U(0,1)$ random variable?

b. Find the probability density function of $Y = \sqrt{X}$.

(12pts)**Problem 2.** Let Z be a standard normal random variable, i.e., it has probability density function

$$f(z) = \frac{1}{\sqrt{2\pi}} \exp(-z^2/2), \quad -\infty < z < \infty$$

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a. What is $P[-3 < Z < 1]$?

b. What is the distribution of $Y = Z^2$? In other words, what is the density of Y?

c. Let X_1, X_2 be a random sample from the standard normal distribution. What is the distribution of $W = X_1^2 + X_2^2$?

Problem 3. Answer the following questions.
(A) Let $X_1, X_2, \cdots, X_{26}$ be independent exponential ($\lambda = 1$) random variables. What is the **exact** distribution of

$$G = \frac{X_1 + X_2 + \cdots + X_{26}}{26}$$

(a)Hypergeometric(26,26, 6). (b) Gamma ($\alpha = 26, \lambda = 26$). (c) Gamma($\alpha = 26, \lambda = 1$). (d)Chi-squared(26). (e) none of the above

(B) Let $X_1, X_2, \cdots, X_{26}$ be independent Geometric ($p = \frac{1}{4}$) random variables. What is the **exact** distribution of

$$G = X_1 + X_2 + \cdots + X_{26}$$

(a)Hypergeometric(100, 50, 6). (b) Negative Binomial ($r = 26, p = \frac{1}{4}$). (c) Negative Binomial ($r = 13, p = \frac{1}{4}$). (d)Chi-squared(26). (e) none of the above

Problem 4. Let $Y = X_1 + X_2 + \cdots + X_{49}$ be the sum of a random sample of size 49 from the distribution whose p.d.f is

$$f(x) = \begin{cases} 4x^3, & 0 < x < 1 \\ 0, & \text{otherwise.} \end{cases}$$

Approximate $P[-14.4 < Y < 28.8]$.
(Hint; use the central limit theorem: $\bar{X} \sim N(\mu, \sigma^2/n)$.)