1- Probability

- random experiment
- Sample space, S

examples : 1) roll a fair die

2) roll two fair dice, consider the sum

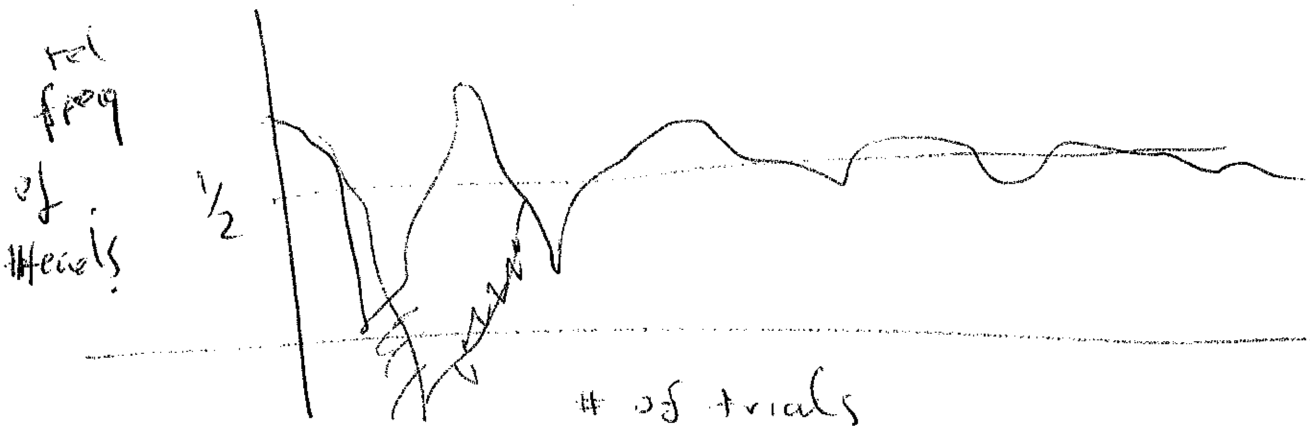
3) flip a fair coin ~~until~~ 2 times

4) flip a coin until you see Heads for the first time.

Interpretation of probability

perform an experiment over and over again
count number of occurences of one type
(Heads) then

$$p(H) = \lim_{n \to \infty} \frac{\#(H) \text{ in } n \text{ flips}}{n}$$



1.2 : Event $A \subseteq S$ (2)

insert 2.1 - 2.2 from STAT 430 notes (summer 08)

- event notations,
- intersection, unions, subset . . .
- mutually exclusive, exhaustive . . .
- Venn diagrams
- tree diagrams . . .

· probability

$$P : \{\text{subsets of } S\} \longrightarrow [0,1]$$

a) $P(A) \geq 0$

b) $P(S) = 1$

c) $A_1, \ldots A_k \ldots$ are mutually exclusive events, i.e $A_i \cap A_j = \phi$ $i \neq j$ then

$$P(A_1 \cup \ldots A_k) = P(A_1) + \ldots P(A_k)$$

for each positive integer k

and

$$P(A_1 \cup A_2 \cup \ldots) = P(A_1) + P(A_2) \ldots$$

for an infinite, but countable, number of events

End class 1

other rules

1) Complementary prob.

$$P(A) = 1 - P(A')$$

example A = you need at ~~most~~ (least) 1 trial to get the first success

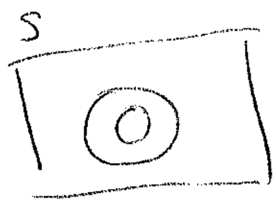
$$P(A^{B}) = 1 - P(A')$$
$$= 1 - P(H) = 1 - \frac{1}{2} = 1/2$$

⋮

2) $P(\phi) = 0$

3) $A, B \subseteq S$; $A \subseteq B \Rightarrow P(A) \leq P(B)$

proof by picture



4) $A \subseteq S$: $0 \leq P(A) \leq 1$

5) $P(A \cup B) = P(A) + P(B) - P(A \cap B)$

Example: Problem $\boxed{1.2-\cancel{8}}$ (1.2-5) — Eighth edition and some variations

HW: (1.2) # 1–4, 6, 7

rule 6 : $A, B, C \subseteq S$

$$P(A \cup B \cup C) = P(A) + P(B) + P(C) - P(A \cap B) - P(A \cap B) - P(B \cap C) + P(A \cap B \cap C)$$

proof : (picture) Venn diagram.

* "equally likely" outcome experiment

1) toss a fair coin
2) " " " die.

so next chapter is motivated . . .

HW: (1,2) ~~9~~ #9, #11, #17

<u>Example</u>: suppose $S = \{w_1, w_2, \dots w_N\}$ has N distinct elements ("N distinct outcomes of the experiment"). One way of assigning probabilities to every subset of S is to just let

$$P(A) = \frac{|A|}{|S|} = \frac{|A|}{N}$$

where $|A|$ denotes the number of elements of A.

This <u>function</u> (P) verifies all the rules of a probability function.

$$p(\{w_i\}) = \frac{1}{N}.$$

thus, these are "equally likely outcomes".

<u>Example</u>: flip two fair coins.

$$S = \{H_1H_2, H_1T_2, T_1H_2, T_1T_2\}$$

"suppose" the outcomes are equally likely.

$$\begin{aligned} P(\{H_1\}) &= P(\{H_1H_2\} \cup \{H_1T_2\}) \\ &= P(\{H_1H_2\}) + P(\{H_1T_2\}) \\ &= \frac{1}{4} + \frac{1}{4} = 1/2. \end{aligned}$$

<u>check</u>: $p(H_1) = p(H_2) = p(T_1) = p(T_2)$

End Lecture 2

1.3 : Methods of enumeration

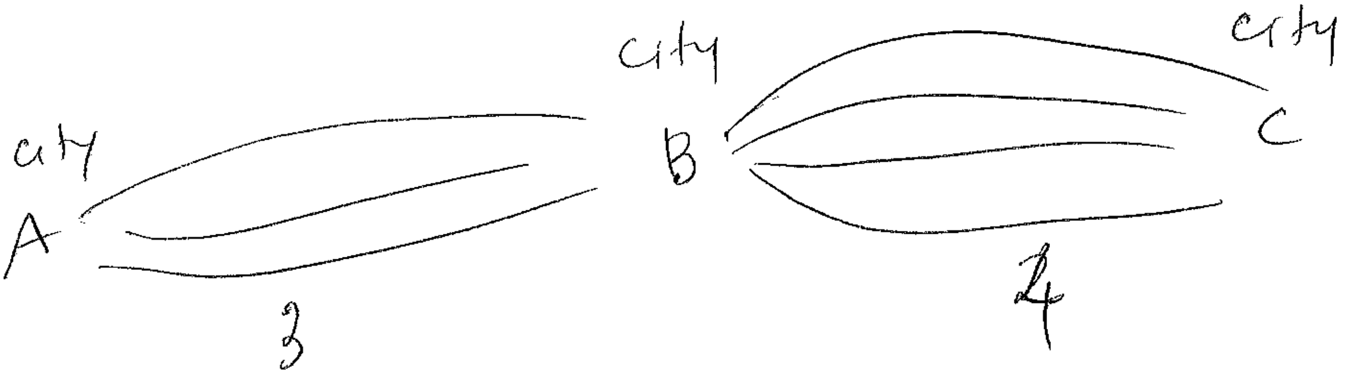
Many questions in prob are counting-type questions. for example; assign an Auburn resident a random phone #. what is the chance that it is my phone #?

$$\text{Ans} = \frac{1}{\left(\begin{array}{c}\text{total \# of possible}\\ \text{phone \# in Auburn}\end{array}\right)} = \frac{1}{\text{\# of } \square\square\square - \square\square\square\square\text{'s}}$$

$$= \frac{1}{(10 \times 10 \times 10)(10 \times 10 \times 10 \times 10)} = \frac{1}{10^7}.$$

This is the ~~basic principle of counting~~ multiplication ~~rule~~ principle;

" 2 experiments; 1st one has n_1 many possible outcomes, 2nd one n_2. Together there are $n_1 n_2$ many outcomes.



# of different routes from $\underline{A \text{ to } C}$ that passes through B is $\underline{3 \times 4 = 12}$

more generally k experiments each results in n_i possible outcomes for $i = 1, \dots k$.

then total # of outcomes of k experiments is $n_1 \times n_2 \times \dots \times n_k$.

Ex: 5 red balls (labeled 1, 2, ... 5)
4 green balls (labeled 1, 2, ... 4)
pick one of each color

How many ways? $5 \times 4 = 20$.

Example: How many phone numbers on campus beginning with 844?

581 ⎵ ⎵ ⎵ ⎵

experiment #1   #2   #3   #3

$n_1 = 10$ $n_2 = 10$, $n_3 = 10$, $n_4 = 10$

# of phone numbers $= 10 \cdot 10 \cdot 10 \cdot 10$
$= 10^4$.

Example: How many functions defined on n points if the function values are 0 or 1

#1   #2   #3   ...   #n

⎵ ⎵ ⎵ ... ⎵

$n_1 = 2$ $n_2 = 2$ $n_3 = 2$ ... $n_n = 2$

$$\text{Ans} = 2^n = (\underbrace{2 \times 2 \times 2 \times \dots \times 2}_{n\text{-times}})$$

permutations :

There are n-distinct object to be layed out in n positions. How many ways?

$$\underset{\text{1st pos.}}{\overset{n}{\sqcup}} \quad \underset{\text{2nd}}{\overset{n-1}{\sqcup}} \quad \overset{n-2}{\sqcup} \; \cdots \quad \underset{\text{nth positions}}{\overset{1}{\sqcup}}$$

$n(n-1)(n-2)\cdots 1 = n!$ possible arrangements.

—— end (Lecture 3 - August 21-09)

Example : using numbers 1, 2, 3, 4 ; total number of permutatios is $4! = 24$.

But if we allow repetition, # of ways $= 4^4 = 256$.

because each selection can be performed in 4 ways

⇒ if only r positions are filled with objects selected from n distinct objects then

$$_nP_r = n(n-1)(n-2)\cdots(n-r+1)$$
$$= \frac{n!}{(n-r)!}$$

Example : # of ways of selecting a president, a vice president, a secretary and a treasurer in a club consisting of 7 persons is

$$_7P_4 = \frac{7!}{3!} = 7.6.5.4 = 840$$