

STAT 3600 Spring 2010 Final Exam

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Name:

PID:

Instructions

Read each problem carefully. Show all your work. Credit will only be awarded if your work is included. There is a **standard normal table** at the end. There are two pages of **formulae** at the end.

Problem	max-points	Points
1	15	
2	15	
3	20	
4	20	
5	20	
6	20	
7	20	
8	20	
9	20	
10	30	
bonus	20	
total	200+20	

Problem 1. I flip a fair coin. If it is heads I roll a four-sided die (1,2,3,4) and if it is tails I roll a six-sided die.

a. What is the probability the die roll is 2?

b. If the die roll is 2, what is the (conditional) probability that the coin flip showed heads?

Problem 2. An urn contains 4 red balls and 3 white balls. Balls are drawn at random one after the other. Every time a ball is drawn, it is **replaced back in the urn along with 1 new balls of the same color.** (For example; If you pick a white ball in the first selection, for the second selection you have 5 white balls and 5 red balls.)

a. Compute the probability that a red ball is drawn on the first draw.

b. Compute the probability that a red ball is drawn on the second draw.

Problem 3. Let X have the following probability density function

$$f(x) = \begin{cases} x, & 0 < x < 1 \\ 2 - x, & 1 < x < 2. \end{cases}$$

a. Find $P(X < 1.7)$.

b. Find the 20th percentile and the 60th percentile of X .

c. Find the **cumulative distribution function** (cdf) F of X .

Problem 4. Let X be a uniform random variable on the interval $(-2, 2)$ and let $Y = X^2$.

a. Find $E(Y)$.

b. Find the probability density function of Y .

c. How can we simulate an observation of X using a $U(0, 1)$, uniform random variable over $(0, 1)$?

d. How can we simulate an observation of Y using a $U(0, 1)$, uniform random variable over $(0, 1)$?

Problem 5. If X has a moment generating function (m.g.f.) as

$$M(t) = (3e^{-t} + 2e^{2t} + 2e^t)/7.$$

(a) Find the probability mass function of X .

(b) Find the mean and variance of X by using the m.g.f., $M(t)$, given above.

c. What is $P[X > 1 | X \geq 0]$?

Problem 6. a. The probability density function of X is given by

$$f(x) = a + bx^2 \quad 0 \leq x \leq 1.$$

If $E(X) = 3/5$ then find a and b .

b. If the moment generating function of a random variable Y is

$$M_Y(t) = e^{5t+200t^2}$$

Find the probability density function of Y .

Problem 7. Let X be a $N(0, 100)$ normal random variable. That is the pdf of X is

$$f_X(x) = \frac{1}{10\sqrt{2\pi}} \exp(-x^2/200), \quad -\infty < x < \infty.$$

a. Find $P[-10 < X < 5.5]$

b. Find the probability density function of

$$W = (X/10)^4.$$

Problem 8. (2pts)a. Indicate how to show that sum of independent normal random variables is a normal random variable using moment generating functions. (Hint: If $X \sim N(\mu, \sigma^2)$, then $M_X(t) = \exp(t\mu + t^2\sigma^2/2)$)

b. You have 144 light bulbs whose lifetimes are independent normal random variables with mean 100 (hours) and standard deviation 5 (hours). Find the probability that your 144 light bulbs together live for at least 14402 hours.

(c) Find a number M such that, the total lifetime of 144 light bulbs is less than M (hours) with the probability 0.02. (Hint: In other words, $P[X_1 + X_2 + \cdots + X_{144} < M] = 0.02$, in other words this is the 2th percentile of the sum)

Problem 9. Let X have a probability density function

$$f(x) = \begin{cases} 2x, & 0 < x < 1 \\ 0, & \text{otherwise.} \end{cases}$$

a. Let $Y = X_1 + X_2 + \cdots + X_{36}$ be the sum of a random sample of size 36 from the distribution whose p.d.f is $f(x)$. Approximate $P[-14.4 < Y < 28.8]$. (Hint; use the central limit theorem: $\bar{X} \sim N(\mu, \sigma^2/n)$.)

b. Let $Y_n = X_1 + X_2 + \cdots + X_n$ be the sum of a random sample of size n from the distribution whose p.d.f is $f(x)$. Discuss

$$\lim_{n \rightarrow \infty} P[-14.4 < Y_n < 28.8].$$

Problem 10. Answer the following questions.

(A) Select 3 integers from $\{1, 2, 3, \dots, 49, 50\}$ **without replacement**. Let X be the number of integers less than or equal to 5 in the selection. How is X distributed?

(a) Hypergeometric(50, 25, 6). (b) Geometric (0.5). (c) Hypergeometric (5,3,50). (d) Binomial (3, 0.1). (e) none of the above

(B) Select 3 integers from $\{1, 2, 3, \dots, 49, 50\}$ **with replacement**. Let Y be the number of integers less than or equal to 5 in the selection. How is Y distributed? (a) Hypergeometric (5,3,100). (b) Binomial (50, 0.1). (c) Poisson(0.5). (d) Binomial(3,0.1). (e) none of the above

(C) If the Moment generating function of W is

$$M(t) = \frac{(0.7e^t)^{10}}{[1 - 0.3e^t]^{10}}, \quad t < -\ln(0.3)$$

How is W distributed?

(a) Hypergeometric(100, 50, 6). (b) Geometric (0.5). (c) negative Binomial(0.7, 10). (d) Binomial (10, 0.7). (e) none of the above

(D) Let X_1, X_2, \dots, X_{25} be independent exponential ($\lambda = 2$) random variables. What is the **exact** distribution of

$$G = \frac{X_1 + X_2 + \dots + X_{25}}{25}$$

(a) Negative Binomial(25, 0.5). (b) Gamma ($\alpha = 25, \lambda = 50$). (c) Gamma($\alpha = 2, \lambda = 50$). (d) normal(25, 25/4). (e) none of the above

Bonus Problem. Let X and Y be independent, standard normal random variables. Recall that their common density is the function

$$f(a) = \frac{1}{\sqrt{2\pi}} \exp(-a^2/2), \quad -\infty < a < \infty$$

Compute the density of the random variable W , where

$$W = \sqrt{X^2 + Y^2}.$$

(Hint: use moment generating functions).