Math 7810-110 Spring 2011 HW #4

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Due Monday April 4, 2011.

Problem 1. If $X_1, \cdots, X_n$ are exchangeable and in L^1, then

$$E\left|\frac{1}{n}\sum_{i=1}^{n} X_i\right| \leq E\left|\frac{1}{n-1}\sum_{i=1}^{n-1} X_i\right|$$

Hint: Use the idea in the beginning of the proof of Theorem 5.6.5. Find a martingale and use Jensen.

Problem 2. Let $M_n, n \geq 0$ be a martingale with $M_n \in L^2$ for each n. Let S, T be bounded stopping times with $S \leq T$. Show that M_S, M_T are both in L^2 and show that

$$E[(M_T - M_S)^2|\mathcal{F}_S] = E[M_T^2 - M_S^2|\mathcal{F}_S]$$

and

$$E[(M_T - M_S)^2] = E[M_T^2] - E[M_S^2]$$

Problem 3. Let $X_1, X_2, \cdots$ be iid exponential random variables with density

$$f(x) = e^{-x}1_{\{x>0\}}.$$

Let $S_n = \sum_{i=1}^{n} X_i$ and let $\tau = \inf\{n : S_n \geq 10\}$

a. Find the distribution of S_τ.

b. Find $E(\tau)$.

Problems from Durrett 4th edition. 4.1.3, 4.1.7, 5.6.3, 5.6.5, 5.7.1, 5.7.2, 5.7.3