Math 7800-110 Fall 2010 Exam #2

Instructor: Erkan Nane

Due Wednesday November 17, 2010.

Problem 1. Suppose that $X_1, X_2, \cdots$ are non-negative random variables with $E(X_k) = 1$ and $X_k \to c$ in probability for some constant c. What are the possible values of c?

Problem 2. Suppose that $X_1, X_2, \cdots$ are non-negative random variables with $E(X_k) = 1$. Suppose also that $E(X_k^2) \to 1$ as $k \to \infty$. Can we conclude that $X_k \to X$ in probability, for some random variable X?

Problem 3. Let $Y_1, Y_2, \cdots$ be independent Uniform $[0, 2]$ random variables. That is, their density is $f(y) = \frac{1}{2} 1_{\{0<y<2\}}$. Let $W_n = \prod_{k=1}^{n} Y_k$. Prove that $W_n \to 0$ a.s. (Hint; Use SLLN

Problem 4. Suppose that the random variables $X_1, X_2, \cdots$ are defined on the same probability space and let $Y_n = \sup_{k \geq n} |X_k|$. Show that the following two statements are equivalent:

(i) $X_n \to 0$ a.s. as $n \to \infty$

(ii) $Y_n \to 0$ in probability as $n \to \infty$

Problem 5. Suppose that $A_1, A_2, \cdots$ are events in a probability space, and that these are "negatively dependent":

$$P(A_i \cap A_j) \leq P(A_i)P(A_j), \ \forall \ i \neq j.$$

Show that $P(A_n \ i.o. \ ) = 1$ if $\sum_{n=1}^{\infty} P(A_n) = \infty$.

Problem 6. Suppose that $X_1, X_2, \cdots$ are iid random variables, and let $r > 0$. Show that the following are equivalent:

(i) $E(|X_1|^r) < \infty$;

(ii)$\sum_{n=1}^{\infty} P(|X_n| > n^{1/r}\epsilon) < \infty \ \ \forall \ \epsilon > 0$;

(iii) $P(|X_n| > n^{1/r}\epsilon \ i.o.) = 0 \ \ \forall \ \epsilon > 0$;

(iv) $\frac{X_n}{n^{1/r}} \to 0$ a.s. as $n \to \infty$.

Problem 7. Let $X_1, X_2, \cdots$ be iid random variables with $E|X_1| < \infty$. Show that

$$\frac{\max_{1 \leq k \leq m} X_k}{n} \to 0 \ \ a.s.$$

Exercises from the book(4th edition, Durrett's book): #2.2.3, #2.3.11, # 3.2.3, #3.2.13