

## Math 5/6680 Spring 2009 Quiz #7

Name:

PID:

(6pts)**Problem 1.** Consider a Markov chain with state space  $\{1, 2, 3, 4, 5\}$  and transition matrix

$$\begin{pmatrix} 0.4 & 0.3 & 0.3 & 0 & 0 \\ 0 & 0.5 & 0 & 0.5 & 0 \\ 0.5 & 0 & 0.5 & 0 & 0 \\ 0 & 0.5 & 0 & 0.5 & 0 \\ 0 & 0.3 & 0 & 0.3 & 0.4 \end{pmatrix}$$

**a.** Is this chain irreducible? If not, what are the communication classes?

**b.** Use the fact that “If a state  $x$  communicates with a state  $y$ , but  $y$  does not communicate with  $x$ , then  $x$  is a transient state” to determine transient and recurrent states.

(12pts)**Problem 2.** Consider a Markov chain with state space  $\{1, 2, 3\}$  and transition matrix

$$\begin{pmatrix} 0 & 1/2 & 1/2 \\ 0 & 2/3 & 1/3 \\ 1/3 & 2/3 & 0 \end{pmatrix}$$

- a. Is the chain irreducible? Is it aperiodic? Verify.
- b. Assume that the chain starts at 1. What is the expected number of time intervals until the chain is in state 1 again?
- c. Suppose that the chain starts at 2, what is the expected number of steps to reach state 1?

(12pts)**Problem 3.** Let  $X_n$  be a Markov chain on state space  $\{0, 1, 2, 3\}$  with transition matrix

$$\begin{pmatrix} 1 & 0 & 0 & 0 \\ 0.6 & 0 & 0.4 & 0 \\ 0 & 0.6 & 0 & 0.4 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

(3pts)**a.** What are the transient states? What are the recurrent states?

(3pts)**b.** What is the matrix  $Q$  that corresponds to transient classes?

(3pts)**c.** Starting from state 1 what is the expected number of visits to state 2 before absorption?

(3pts)**d.** Starting from state 1 What is the expected time until absorption ?