

Math 2650

A. J. Meir

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Consider the equation

$$\frac{dx}{dt} + p(t)x = f(t).$$

This is a linear equation. We can always solve such equations analytically (at least get an integral representation for the solution).

To solve such an equation find an integrating factor $\mu(t)$, such that when we multiply the equation by the integrating factor

$$\frac{\mu(t) dx}{dt} + \mu(t) p(t) x = \mu(t) f(t)$$

we get

$$\frac{d}{dx}(\mu(t) x(t)) = \mu(t) f(t).$$

Note that since

$$\frac{d}{dx}(\mu(t) x(t)) = x(t) \frac{d\mu}{dt} + \mu(t) \frac{dx}{dt}$$

we must have

$$\frac{d\mu}{dt} - \mu p = 0$$

this is a separable equation which has a solution

$$\mu(t) = e^{\int p(t) dt}.$$

Hence

$$\mu(t) x(t) = \int \mu(t) f(t) dt + C$$

or

$$x(t) = \frac{\int \mu(t) f(t) dt + C}{\mu(t)}$$

Examples:

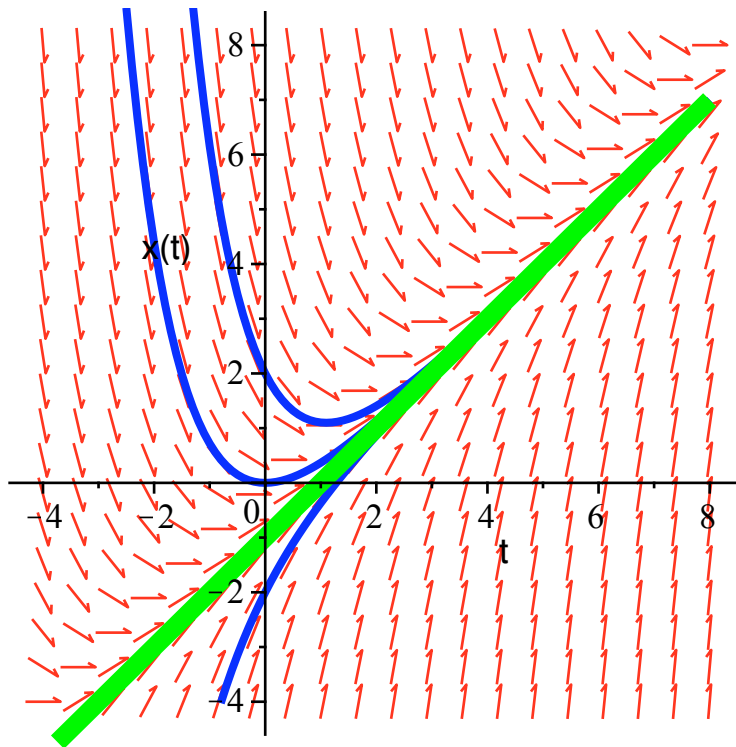
[> **restart:with(DEtools):with(plots):**

Consider the equation


```

> sol:=DEplot(D(x)(t)+x(t)=t,x(t),t=-4..8,x=-4..8,[[x(0)=-2],[x(0)=0],[x(0)=2]],linecolor=blue,stepsize=0.1):
> ss:=plot(t-1,t=-4..8,x=-4..8,color=green,thickness=7):
> display(sol,ss);

```



Consider the equation

$$\frac{dx}{dt} + x = \sin(t)$$

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> mu:=exp(int(1,t));

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$$\mu := e^t$$

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> int_mu_f:=int(sin(t)*exp(t),t);

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$$\text{int_mu_f} := -\frac{1}{2} \cos(t) e^t + \frac{1}{2} \sin(t) e^t$$

```

> dsolve(D(x)(t)+x(t)=sin(t),x(t));

```

$$x(t) = -\frac{1}{2} \cos(t) + \frac{1}{2} \sin(t) + _C1 e^{-t}$$

```

> sol:=DEplot(D(x)(t)+x(t)=sin(t),x(t),t=-1..15,x=-2..2,[[x(0)=-2],[x(0)=0],[x(0)=2]],linecolor=blue,stepsize=0.1):
> ss:=plot((-1/2)*cos(t)+(1/2)*sin(t),t=-1..15,x=-2..2,color=green,thickness=7):
> display(sol,ss);

```

