# Math 2650

*A. J. Meir*



Consider the equation

$$\frac{dx}{dt} + p(t)\,x = f(t).$$

This is a linear equation. We can always solve such equations analytically (at least get an integral representation for the solution.

To solve such an equation find an integrating factor $\mu(t)$, such that when we multiply the equation by the integrating factor

$$\frac{\mu(t)\,dx}{dt} + \mu(t)\,p(t)\,x = \mu(t)\,f(t)$$

we get

$$\frac{d}{dx}(\mu(t)\,x(t)) = \mu(t)\,f(t).$$

Note that since

$$\frac{d}{dx}(\mu(t)\,x(t)) = x(t)\,\frac{d\,\mu}{dt} + \mu(t)\,\frac{dx}{dt}$$

we must have

$$\frac{d\,\mu}{dt} - \mu\,p = 0$$

this is a seperable equation which has a solution

$$\mu(t) = e^{\int p(t)\,dt}.$$

Hence

$$\mu(t)\,x(t) = \int \mu(t)\,f(t)\,dt + C$$

or

$$x(t) = \frac{\int \mu(t)\,f(t)\,dt + C}{\mu(t)}$$

Examples:

```
> restart:with(DEtools):with(plots):
```

Consider the equation

$$\frac{dx}{dt} + x = 0$$

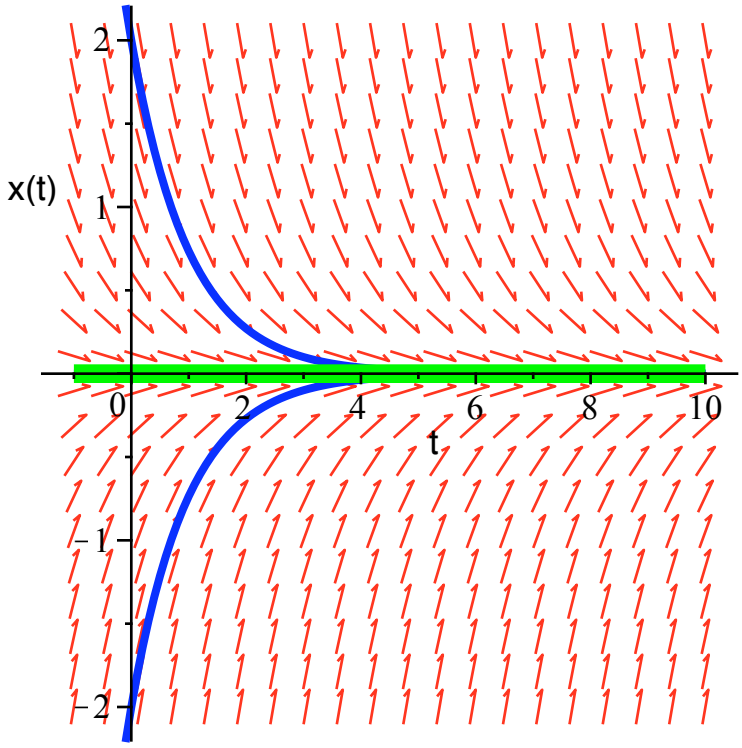
```
> mu:=exp(int(1,t));
```

$$\mu := e^{t}$$

```
> dsolve(D(x)(t)+x(t)=0,x(t));
```

$$x(t) = \_C1\, e^{-t}$$

```
> sol:=DEplot(D(x)(t)+x(t)=0,x(t),t=-1..10,x=-2..2,[[x(0)=-2],
  [x(0)=0],[x(0)=2]],linecolor=blue,stepsize=0.1):
> ss:=plot(0,t=-1..10,x=-2..2,color=green,thickness=7):
> display(sol,ss);
```




Consider the equation

$$\frac{dx}{dt} + x = t$$

```
> mu:=exp(int(1,t));
```

$$\mu := e^{t}$$

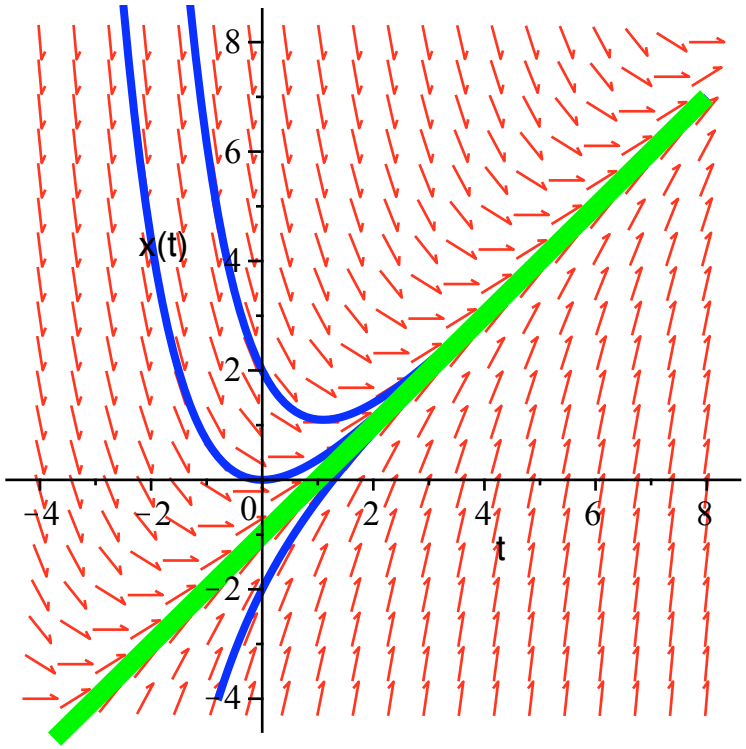
```
> int_mu_f:=int(t*exp(t),t);
```

$$int\_mu\_f := (-1 + t)\, e^{t}$$

```
> dsolve(D(x)(t)+x(t)=t,x(t));
```

$$x(t) = -1 + t + \_C1\, e^{-t}$$

```
> sol:=DEplot(D(x)(t)+x(t)=t,x(t),t=-4..8,x=-4..8,[[x(0)=-2],[x
  (0)=0],[x(0)=2]],linecolor=blue,stepsize=0.1):
> ss:=plot(t-1,t=-4..8,x=-4..8,color=green,thickness=7):
> display(sol,ss);
```




Consider the equation

$$\frac{dx}{dt} + x = \sin(t)$$

```
> mu:=exp(int(1,t));
```

$$\mu := e^{t}$$

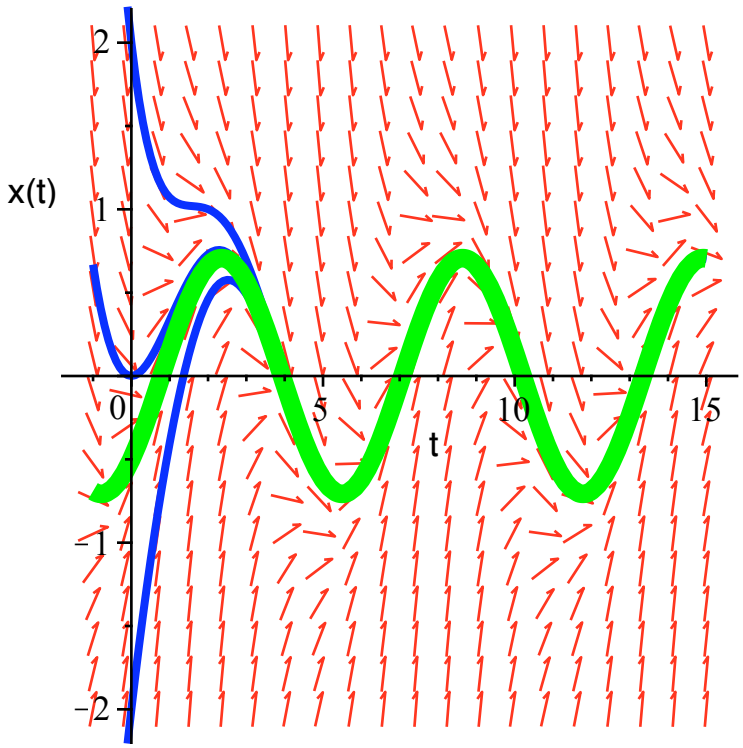
```
> int_mu_f:=int(sin(t)*exp(t),t);
```

$$int\_mu\_f := -\frac{1}{2}\cos(t)\,e^{t} + \frac{1}{2}\sin(t)\,e^{t}$$

```
> dsolve(D(x)(t)+x(t)=sin(t),x(t));
```

$$x(t) = -\frac{1}{2}\cos(t) + \frac{1}{2}\sin(t) + \_C1\,e^{-t}$$

```
> sol:=DEplot(D(x)(t)+x(t)=sin(t),x(t),t=-1..15,x=-2..2,[[x(0)=
  -2],[x(0)=0],[x(0)=2]],linecolor=blue,stepsize=0.1):
> ss:=plot((-1/2)*cos(t)+(1/2)*sin(t),t=-1..15,x=-2..2,color=
  green,thickness=7):
> display(sol,ss);
```


x(t)
2
1
0
5
10
15
t
−1
−2