Special Session on

"Magnetohydrodynamics — Analysis and Applications"

Third World Congress of Nonlinear Analysts, July 19–26, 2000, Catania, Sicily

Organizers

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Summary

Magnetohydrodynamics (or MHD) is the theory of the macroscopic interaction of electrically conducting fluids with magnetic fields. It is of importance in connection with many engineering problems as well as in geophysics and astronomy. Much research has been devoted to the physical modeling and computational simulation of MHD-dominated processes, but there is still a shortage of rigorous analytical and numerical methods. Future progress will likely depend on an intensified crossdisciplinary dialogue between analysts, computationalists, physicists, and engineers in the field. Continuing the tradition established at WCNA-1996, this special session will attempt to contribute to this dialogue by bringing together researchers from different disciplines concerned with mathematical and computational questions of MHD. Speakers will be encouraged to emphasize crossdisciplinary aspects of their research, so that the session should appeal to a broad audience.

Session Program and Schedule

Date: Thursday, July 20, 2000
Time: 8:30 am – 7:00 pm
Room: Aula 36

8:30 **Nagy El-Kaddah**

Department of Metallurgical Engineering, University of Alabama, Box 870202, Tuscaloosa, AL 35487-0202, USA (Email: nelkaddah@coe.eng.ua.edu)

Title of Presentation:

A New Method for Computing Electromagnetic Forces, Fluid Flow, and Temperature Fields in Containerless Melting and Casting Systems

Abstract:

This paper describes a new solution technique for solving coupled electromagnetic, fluid flow and heat transfer equations in an electromagnetically confined molten metal whose shape is not is not known a priori. The key feature of this method is that it does not require regridding the solution domain during the search for the equilibrium meniscus shape. It is based on the solution of these equations in a fixed computational domain using a variable non-orthogonal coordinate transformation. Within this framework, the electromagnetic field was computed using the technique of mutual inductance, while the temperature and velocity fields in the slab were calculated using the control volume technique. This method was applied to simulate melt containment, fluid flow and heat transfer phenomena in electromagnetic casters and the magnetic suspension melting process. Excellent agreement was found between the computed results and experimental measurements regarding the shape of the melt free surface, temperature distribution in the metal, and the rate of heating and melting of the metal. It is suggested that this versatile methodology can be used with confidence in future process models aimed at providing comprehensive knowledge of the various phenomena involved in electromagnetic confinement systems. (This is joint work with J. R. Bhamidipati.)

9:30 Rachid Touzani

Laboratoire de Mathématiques Appliquées, Université Blaise Pascal (Clermont-Ferrand 2),
63177 Aubiere cedex, France (Email: touzani@ucfma.univ-bpclermont.fr)

Title of Presentation:

Asymptotic Behaviour of the Inductance Coefficient for Thin Conductors in Eddy Current Models

Abstract:

We show a decomposition property of the potential as a sum of a singular potential and a finite energy one. The singular part is explicitly calculated. This decomposition allows to describe the asymptotic behaviour of the inductance coefficient for a thin toroidal inductor. This problem constitutes a first step in deriving mathematical models for eddy current problems where thick conductor bodies and thin toroidal circuits are involved.

10:30 Break

11:00 Maurice H.P.M. van Putten

Department of Mathematics, Massachusetts Institute of Technology, Cambridge, MA 02139, USA
(Email: mvp@math.mit.edu)

Title of Presentation:

Electron-Positron Outflow from Black Holes

Abstract:

Gamma-ray bursts (GRBs) appear as the brightest transient phenomena in the Universe. The observed emission is well described by the internal-external shock fireball model, produced by ultra-relativistic outflow into a wind or the interstellar medium. The origin of the outflow is most probably associated with a black hole, either newly formed or pre-existing, surrounded by a magnetized torus. I shall discuss intermittency and leptonic outflow in the black hole-torus state, powered by the spin of the black hole. Calculations show consistency with the GRB data. It is predicted that the GRB and the gravitational wave emissions are anti-correlated, which may be verified by the upcoming Caltech-MIT and the European VIRGO gravitational wave observatories.

12:00 Norbert Seehafer

Institut für Physik, Universität Potsdam, PF 601553, D-14415 Potsdam, Germany
(Email: seehafer@agnld.uni-potsdam.de)

Title of Presentation:

Fluid Helicity and Dynamo Effect

Abstract:

The generation and maintenance of magnetic fields by the motion of electrically conducting fluids is the subject of dynamo theory. Using the incompressible magnetohydrodynamic equations, we have studied the dynamo problem numerically. The necessary energy input into the system was either modeled by different kinds of external-forcing terms in the Navier-Stokes equation or fully selfconsistently by thermal convection in a fluid layer heated from below. If the fluid motion is capable of dynamo action, the dynamo effect appears in the form of a phase transition or bifurcation at some critical Reynolds number. Both the dynamo bifurcation and the subsequent bifurcations that occur when the Reynolds number is further raised were studied, including the transition to chaotic states. Special attention was paid to the helicity of the flow as well as to the symmetries of the system and symmetry breaking in the bifurcations. The magnetic field tends to be accumulated in special regions of the flow, for instance in the vicinity of stagnation points. In order to understand these accumulation effects, we have also studied the chaotic Langrangian dynamics of passive tracers.

1:00 Lunch

2:00 Leaf Turner

T-3, MS-B216, Theoretical Division, Los Alamos National Laboratory, Los Alamos, NM 87545, USA (Email: tleaf@lanl.gov)

Title of Presentation:

Elucidation of Inhomogeneous Turbulence: Synergism between Magnetohydrodynamics and Fluid Dynamics

Abstract:

Magnetic fields evolve and persist in media long past the time when they naively would have been expected to dissipate resistively. This unexpected phenomena, first pointed out by Cowling in 1934, continues to inspire much research. A turbulent "MHD dynamo" mechanism is believed to be responsible for the sustainment of the field, whether it be the field in astrophysical bodies, such as the Earth, or in laboratory fusion experiments, such as the reversed-field pinch (RFP). The principal reason believed to be responsible for this sustainment is the multidimensional nature of turbulence. When we first attempted an analysis of this phenomenon, we used a statistical approach for an inviscid perfectly conducting magnetofluid employing the concept of an absolute equilibrium ensemble based on the conserved quantities of energy and magnetic helicity. Although this approach led qualitatively and naturally to the "reversed" magnetic field observed in RFP's, it also had the characteristic pathology of continuum statistical treatments: the Rayleigh-Jeans ultraviolet catastrophe. Avoiding this pathology requires the development of a formalism for treating the dynamical effects of viscosity and resistivity on the statistical dynamics of a physical magnetofluid. We are in the process of developing such a formalism by adapting well-known closures used in theoretical treatments of fluid turbulence. In the process, however, we have cross-fertilized fluid turbulence with our predilection from fusion physics for treating bounded, inhomogeneous systems. We have developed a highly compact and descriptive formalism that allows us to broach the theoretically formidable morass of inhomogeneous turbulence for exploration. Our formalism has two novel aspects: (a) an adaptation of helicity basis functions to represent an arbitrary incompressible channel flow and (b) the invocation of a hypothesis of random phase. A result of this compact formalism is that the mathematical description of inhomogeneous fluid turbulence looks much like that of the homogeneous case — at the moment, the most rigorously explored terrain in fluid turbulence research. As a result, we can explore the effect of boundaries on important quantities as the gradients of mean flow, mean pressure, triple-velocity correlations and pressure velocity correlations, all of which vanish when one makes the conventional, but artificial, assumption that the turbulence is statistically spatially uniform. Under suitable parity-violating conditions on the energy spectrum, we have predicted that a mean flow gradient can develop even when none is initially present. However, in order to demonstrate realizability in the two-fluid-like case of MHD turbulence, one must demonstrate that (a) the turbulent fluid energy spectrum remains positive at all times, (b) that the turbulent magnetic energy spectrum remains positive at all times, (c) and that the cross-helicity, critical to the turbulent MHD dynamo, satisfies a Schwarz inequality. To do so, one must show that the three evolution equations of the three spectra obtained from the closure yield evolving spectra that always satisfy these three "realizability" conditions above. In the absence of mean fields, we have done so using the Elsasser field variables, $V \pm B$. (This work was supported by the U.S. Department of Energy and Los Alamos National Laboratory under LDRD Project #IP97-018.)

2:45 Juan Francisco Padial Molina

Departamento de Matemática Aplicada, E.T.S.A., Universidad Politécnica de Madrid, 28040 Madrid, Spain (Email: jfpadial@aq.upm.es)

Title of Presentation:

On a Parabolic Model Arising in Stellarator Devices

Abstract:

We consider a 2D free-boundary problem arising in the transient regime of a magnetically confined plasma in a Stellarator device. From ideal 3D MHD systems, taking into account the Ohm and Faraday laws and the averaging arguments of Hender and Carreras (*Phys. Fluids*, 1984), we obtain a two-dimensional evolution model. By using some arguments on the characteristic times for the involved phenomena, in particular, by assuming that all processes are slow (i.e., quasi-stationary processes), the model becomes stationary in the plasma region, while it remains an evolution problem in the vacuum region. Thus the obtained model is defined by a nonlinear elliptic-parabolic partial differential equation. — Under some regularity hypotheses for the initial data we give a result of existence and regularity of solutions for the evolution problem. The main idea is to approximate the original elliptic-parabolic problem by a family of uniformly parabolic problems, which do not become stationary in the plasma region. By using a Galerkin argument we find solutions for these problems. We use some technical approach in order to pass to the limit in such problems. As far as we know, this is the first mathematical attempt to study the transient regime of a Stellarator. (This is joint work with J. I. Díaz.)

3:30 **Gerhard O. Ströhmer**
Department of Mathematics, MacLean Hall, University of Iowa, Iowa City, IA 52242-0001, USA
(Email: strohmer@math.uiowa.edu)

Title of Presentation:

About the Equations of Non-stationary MHD on Domains with Non-smooth Boundaries

Abstract:

We obtain weak solutions for the equations of magneto-hydrodynamics which are regular for almost all times. If these solutions are small in a suitable sense they are regular for all times. The conductivity, and to a lesser extent the magnetic permeability, are allowed to vary discontinuously, and the domains in question do not need to be smooth.

4:15 Break

4:45 **Paul G. Schmidt**
Department of Mathematics, Parker Hall, Auburn University, AL 36849-5310, USA
(Email: pgs@math.auburn.edu)

Title of Presentation:

Mathematical Modeling of Electromagnetically and Thermally Driven Liquid-Metal Flows

Abstract:

Electromagnetically and thermally driven flows of viscous incompressible fluids arise in numerous engineering applications, specifically in the metals processing industry. Such flows are governed by the Navier-Stokes equations, Ohm's law, Maxwell's equations, and an energy balance equation in terms of temperature. A comprehensive model must incorporate many different energy and momentum transport mechanisms in the fluid and must account for the electromagnetic interaction of the fluid with the surrounding media. In this talk we will discuss modeling aspects and analytical results relating to the well-posedness of the ensuing system of equations. (This is joint work with A. J. Meir.)

5:30 **A. J. Meir**
Department of Mathematics, Parker Hall, Auburn University, AL 36849-5310, USA
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Title of Presentation:

Finite-Element Simulation of Electromagnetically and Thermally Driven Liquid-Metal Flows

Abstract:

We describe a novel approach to the mathematical modeling and computational simulation of fully three-dimensional, electromagnetically and thermally driven liquid-metal flow. The phenomenon is governed by the Navier-Stokes equations, Maxwell's equations, Ohm's law, and the heat equation, nonlinearly coupled via Lorentz and electromotive forces, buoyancy forces, and convective and dissipative heat transfer. Employing the current density rather than the magnetic field as the primary electromagnetic variable, it is possible to avoid artificial or highly idealized boundary conditions for electric and magnetic fields and to account exactly for the electromagnetic interaction of the fluid with the surrounding media. A finite-element method based on this approach is used to simulate the flow of a metallic melt. (This is joint work with P. G. Schmidt.)

6:15 Richard Jordan

Department of Mathematical Sciences, Worcester Polytechnic Institute, Worcester, MA 01609-2280, USA (Email: rjordan@wpi.edu)

Title of Presentation:

A Statistical Equilibrium Model of Coherent Structures in the Nonlinear Schrödinger Equation

Abstract:

We present a mean-field statistical model of self-organization in a generic class of focusing, non-integrable, nonlinear Schrödinger (NLS) equations. Such equations provide natural prototypes for nonlinear dispersive wave turbulence in fluids, plasmas, and optics. The main conclusion of the theory is that the statistically preferred state for such a system is a macroscopic solitary wave coupled with fine-scale turbulent fluctuations. The solitary wave is a minimizer of the Hamiltonian for a fixed particle number (or L^2 norm squared), and the kinetic energy contained in the fluctuations is equipartitioned over wavenumbers. Numerical simulations of the NLS equation are performed to test the predictions of the statistical model. It is demonstrated that the model accurately describes both the coherent structure and the spectral properties of the solution of the NLS system in the long-time limit. Time permitting, we will discuss the extension of the statistical theory to two-dimensional magnetofluid turbulence, where the formation of coherent structures is also observed.